Calculations of Relative Velocity in Abrasive Wear Systems Employing Laps of Constant Radius of Curvature

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A rigid spherical lap and rigid part in extended contact can only be spheres coincident to within the dimension of the intervening abrasive. Their only motion with respect to each while maintaining contact are their individual rotations. If the axes of these two rotations are noncoincident, they define a plane. If the axes are moving with respect to each other, this movement can be regarded as a rotational vector in a direction orthogonal to the plane defined by the two axes above, and this vector can be added to either of the two rotations described above or apportioned between them to define a reference frame. This demonstrates that at any moment there are actually only two independent rotations. For those familiar with vector algebra, the velocity of either sphere at any point on its surface can be described as the vector or cross product of its rotational vector with the radius vector of the point, i.e. $\vec{V} = \vec{\omega} \times \vec{R}$. This discussion of the changing relationship between the rotational axes is also a precise description of a part stroked across a lap.

Friction, which can be shown to be Coulomb friction, will tend to bring any part in continuous contact with an abrasive lap into rotational synchronization with that component of the lap rotation passing through the centroid or center of gravity (CG) of the contact area, provided that the part is free to so rotate. However, friction and inertial effects prevent or retard the complete acheivement of such synchronization. In most steady state systems that are frictionally driven, parts tend to rotate slightly slower than the lap component passing through the centroid of contact area. The relative motion leading to wear is then almost entirely due to the component of lap rotation orthogonal to the radius passing through the centroid of contact area. Relative velocity tends to a slight maximum in the region of this centroid for finite radii of curvature.

Simultaneous translations of the centroid and rotations about it produce wear in a circular zone about the centroid that in plane systems (infinite radius of curvature) may be summed for averaging purposes as an elliptic integral of the second kind. This is quite general and usually convenient. The integrand of the averaging integral is a radical whose radicand is the square of the relative velocity. In a plane systems the radicand is the form

$$
\begin{aligned}
V^2 &= A^2 + B^2 + 2AB\cos x && \text{where } A \geq B \\
&= A^2(1 + C^2 + 2C\cos x) && \text{where } 0 \leq C = B/A \\
&= A^2(1 + 2C + C^2 - 2C(1 - \cos x)) \\
&= A^2(1+C)^2\left(1 - \frac{4C}{(1+C)^2}\sin^2\frac{x}{2}\right)
\end{aligned}
$$

The first two factors are usually constants and the third is the radicand function traditionally listed for elliptic integrals of the second kind. Since the *cos* function is even, the limits of the averaging integral need only range from 0 to π at maximum, and obviously for the last form need only range from 0 to $\pi/2$ maximum. The second term in the radicand is usually written as $k^2\sin^2 z$ to emphasize that it must be positive, so that it represents a negative value when preceded by the minus sign. This is a point of considerable importance which we shall return to in setting up limits of integration. The reason for the restrictions on C lies in the convergence criteria for implied series solutions.

Now either sphere or any part of it can represent the lap and the other, the part. We are now going to adopt and describe an all positive layout. We shall assume counterclockwise rotations are positive in the direction of angular increase and that angles are measured from the traditional 3 o'clock line. We shall describe a large flat lap on which is placed a small circular part with its center fixed in place at 3 o'clock with a separation D between centers. Radii measured from lap center are represented by R and those from part center by r. Angles shall be measured from the lines joining the centers and shall be represented by X to a lap radius and by x to a part radius. Angular velocity for the lap shall be Ω and for the part, ω. Any point p within the boundary of the part can be located at the intersection of $\vec{R}$ and $\vec{r}$. The relationship between X and x is defined by the projection of $\vec{R}$ and $\vec{r}$ onto this common height. As mentioned, at steady state in a frictionally-driven system $\Omega \approx \omega$ although linkage friction virtually insures $\omega < \Omega$. If we let $\Delta = \Omega - \omega$ then A as expressed in our equation will represent the larger of either $(r\Delta)$ or $(D\Omega)$ and B will represent the smaller. The argument of the cosine will be either X or x depending on which of $\vec{R}$ or $\vec{r}$ we are holding constant (i.e. do we wish to average lap wear or part wear across the appropriate arc of contact). Now in a driven system, Δ may be negative, as could Ω. In a double-faced lap where thin parts are involved, plates are frequently counter-rotated to reduce drag reactions on the thin part edge. In this case, for calculations for one lap or the other, one value of Ω will be negative. If the sense of either Δ or Ω makes the product C cos(x) become negative, we must change the sign of cos(x) by replacing the dummy variable x with $(\pi - x)$ and this will change the sign of the integral since $d(\pi - x) = -dx$ and this sign may be reversed by interchanging the limits. Thus if we have incomplete integrals initially (i.e. either a lower limit (a) greater than 0 or an upper limit (b) less than π, the new upper and lower limits will be $(\pi - a)$ and $(\pi - b)$ respectively.

The modern double-faced lap is rather complex. In the most sophisticated cases, the lap is an annular zone where at inner and outer radii represented by R_1 and R_2, gear or pin rings rotate at angular velocities Ω_1 and Ω_2, respectively, and the lap surface rotates at Ω_3, we may define an effective part rotation ω and a rotation difference from

$$
\begin{aligned}
E &= \frac{R_2\Omega_2 - R_1\Omega_1}{R_2 - R_1} \\
F &= \frac{R_2\Omega_2 + R_1\Omega_1}{R_2 + R_1} \\
\omega &= E - F \\
\Delta &= \Omega_3 - E
\end{aligned}
$$

With these substitutions, the formulae for this more complex case can be shown to reduce to that derived earlier.

We shall, of course, discuss the single term that describes the velocity of the lap into the finite radius geometry, differentiating spherical from flat lapping, and methods for correcting double-sided lapping surfaces (i.e. concentric spherical errors as well as toroidal through errors).

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